Electric Field and Electric Potential

Remember the unit prefixes m=10^{-3}, $\mu = 10^{-6}$

- **The objective of this assignment is to learn how to calculate the electric field generated by point charges.**

The electrostatic interaction is a long range interaction, in several aspects very similar to the gravitational interaction. This interaction is generated by **electric charges**, just as the gravitational interaction is generated by masses. Note, however, that there are two different kinds of charges, positive and negative, whereas there is only one kind of mass (the mass can only be positive).

A convenient way to describe this interaction is through the **electric field**, $\vec{E}$. A charge distribution creates an electric field in the space. The electric field is a vectorial field. This means that at each point in the space there is a vectorial quantity $\vec{E}$. Note that the electric field is a function of the coordinates.

The SI unit for charge is the coulomb, C. The SI unit for the electric field is N/C.

1 Electric field of a point charge

A point charge q creates at any point P an electric field which is a vector of magnitude

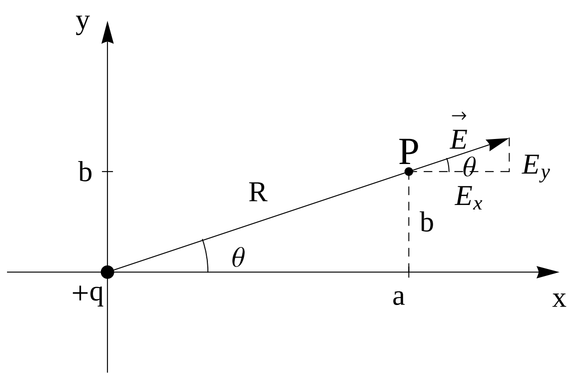
$$|\vec{E}| = k\frac{q}{R^2}\,, \tag{1}$$

where

$$k = 8.99 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2\,. \tag{2}$$

This field at any point P is directed along the line which connects P to the charge, and it is oriented away from the charge q if q is positive, and toward the charge q if q is negative.

Remember that the electric field is a vector. So, often it is necessary to find the components of this field. The general procedure is this: suppose that we want to find



the electric field at P generated by a positive charge $+q$ (see figure). The magnitude of the electric field is

$$|\vec{E}| = k\frac{|q|}{R^2} = k\frac{|q|}{a^2+b^2} \tag{3}$$

Therefore,

$$E_x = |\vec{E}|\cos\theta\,, \qquad E_y = |\vec{E}|\sin\theta\,. \tag{4}$$

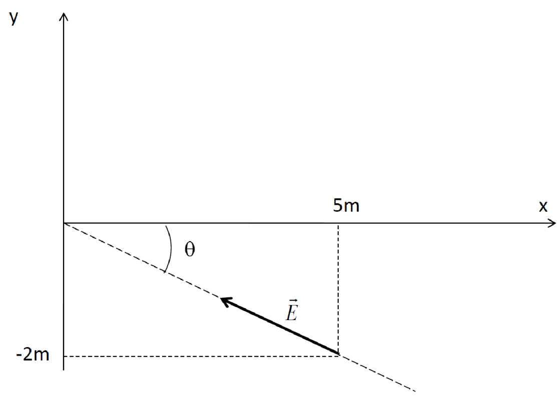
where $\cos\theta = a/R$ and $\sin\theta = b/R$.

Be careful of the signs of the components, since they depend on the choice of your reference frame. *A component is positive if points in the same direction as the arrow of the reference frame.*

Notice that, if the charge were negative, the vector would point toward it, and so its component would have opposite sign with respect to what we wrote above.

- **Example:** A point charge $q = -3$mC, is at the origin of the reference frame. Calculate the electric field generated at the position P=(5m,-2m).

Solution: See the figure The magnitude of the electric field is



$$|\vec{E}| = k\frac{|q|}{R^2} = 8.99\times10^9\text{N}\cdot\frac{\text{m}^2}{\text{C}^2}\times\frac{3\times10^{-3}\text{C}}{29\,\text{m}^2} = 9.3\times10^5\,\text{N/C}\,. \tag{5}$$

Notice also that $\cos\theta = 5/\sqrt{29}$, and $\sin\theta = 2/\sqrt{29}$. Finally, notice that, in the reference frame that we have chosen, the x-component of $\vec{E}$ is negative whereas the y-component is positive

$$E_x = |\vec{E}|\cos\theta = -3.2\times10^4\times\frac{5}{\sqrt{29}}\,\text{N/C} = -8.6\times10^5\,\text{N/C}\ , \tag{6}$$
$$E_y = |\vec{E}|\cos\theta = 3.2\times10^4\times\frac{2}{\sqrt{29}}\,\text{N/C} = 3.4\times10^4\,\text{N/C}\,.$$

- **Problem 1:** A point charge $q = 3$mC, is in the origin of the reference frame. Calculate the electric field generated in position P=(5m,-2m).

- **Problem 2:** A point charge $q = 3$mC, is at the origin of the reference frame. Calculate the electric field generated at the position P=(-3m,-4m).

- **Problem 3:** A point charge $q = 3$mC, is at the position (0,2m). Calculate the electric field generated at the position P=(3m,4m).

2 Superposition Principle

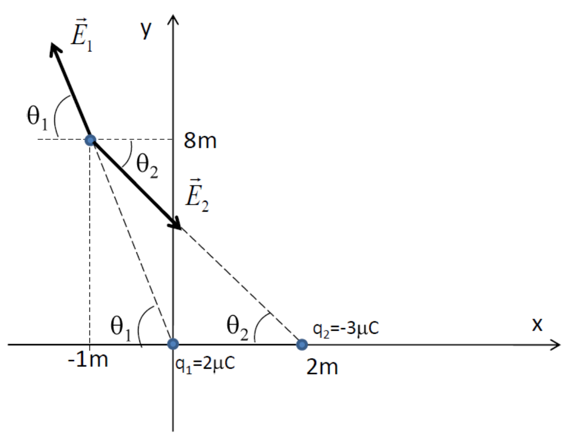
The electric field generated by several charges can be calculated using the **superposition principle**. First you calculate the electric field generated by each single particle, and then sum the results vectorially (x-components with x-components, and y-components with y-components.)

- **Example:**

Suppose you need to calculate the electric field generated by two charges, q_1 and q_2, at a point P. You first calculate the electric field generated by q_1. Suppose that your result is $\vec{E}_1 = (3\text{N}/C, -2\text{N/C})$. Then you calculate the electric field generated by q_2 at P. Suppose that your result is $\vec{E}_2 = (-8\text{N/C}, 12\text{N/C})$. Then, the net electric field at P has components $E_x = 3\text{N/C} + (-8\text{N/C}) = -5\text{N/C}$ and $E_y = -2\text{N/C} + 12\text{N/C} = 10\text{N/C}$. So, $\vec{E} = (-5\text{N/C}, 10\text{N/C})$.

- **IMPORTANT**: Study example 19-5, page 668 in the textbook (PHYSICS, Walker, 4th edition)

- **Guided Problem 4:** A point charge $q_1 = 2\mu\text{C}$ is in (0,0), and a point charge $q_2 = -3\mu\text{C}$ is in (2m,0). Calculate the total electric field in (-1m,8m).



From the figure you can calculate the electric field $\vec{E}_1$ and $\vec{E}_2$ as (pay attention to the signs: you can understand the signs from the directions of the vectors in the figure):

$$\vec{E}_1 = \left(-k\frac{|q_1|}{R_1^2}\cos\theta_1, k\frac{|q_1|}{R_1^2}\sin\theta_1\right) , \quad (7)$$

$$\vec{E}_2 = \left(k\frac{|q_2|}{R_2^2}\cos\theta_2, -k\frac{|q_2|}{R_2^2}\sin\theta_2\right) ,$$

with $R_1 = \sqrt{(1\text{m})^2 + (8\text{m})^2}$, $R_2 = \sqrt{(3\text{m})^2 + (8\text{m})^2}$, $\cos\theta_1 = 1\text{m}/R_1$, $\sin\theta_1 = 8\text{m}/R_1$, $\cos\theta_2 = 3\text{m}/R_2$, $\sin\theta_2 = 8\text{m}/R_2$. After you find $\vec{E}_1$ and $\vec{E}_2$ you should sum the components:

$$\begin{aligned} \vec{E}_x &= \vec{E}_{1x} + \vec{E}_{2x} \,, \\ \vec{E}_y &= \vec{E}_{1y} + \vec{E}_{2y} \,. \end{aligned} \tag{8}$$

Solution: $\vec{E} = (95\,\text{N/C}, -71\,\text{N/C})$

- **Problem 5:** Do the problem above, but with both charges positive $q_1 = q_2 = 1\mu\text{C}$.

3 Electric Potential

A point charge q generates an electric potential V at a point P which is given by

$$V = k\frac{q}{R} \tag{9}$$

where R is the distance from P to the point charge, and q is the charge of the particle. The potential is a scalar, not a vector. The SI units for the potential is volt (V).

Notice that R s always positive, whereas q can be positive or negative.

- **Example:** A point charge $q = 3\text{mC}$ is at (0,2m). Calculate the potential generated at P=(6m, 6m).

Solution: The distance from P to the charge is $R = \sqrt{(4\text{m})^2 + (6\text{m})^2} \simeq 7.2\text{m}$. Therefore, the potential is

$$V = k\frac{q}{R} = \left(8.99 \times 10^9\text{N} \cdot \frac{\text{m}^2}{\text{C}^2}\right) \frac{3 \times 10^{-3}\text{C}}{7.2\text{m}} = 3.75 \times 10^6\text{V}. \tag{10}$$

- **Problem 6:** Calculate the potential generated by a charge $q = 1\mu\text{C}$, 6 m away from the charge. (*Note*: this means that $R = 6\text{m}$.)

The potential generated by 2 charges, is the sum of the potential generated by each one (**superposition principle**).

- **Problem 7:** A point charge $q_1 = 2\mu\text{C}$ is at position (0,0), and a point charge $q_2 = -3\mu\text{C}$ is at position (0,2m). Calculate the total electric potential at position (-1m,8m). (Notice that in this case you don't need to calculate any angle, only R_1 and R_2)